THE TURBULENT DIFFUSION COEFFICIENT OF SUSPENDED PARTICLES

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In [1] a single expression was proposed for the turbulent diffusion coefficient of rather large suspended particles, for which the well-known equation D = N ([2], p. 420) is not fulfilled. Some experimentally verified conditions which limit the applicability of the proposed expression are given below.

NOTATION

ρ_0 and ρ are the densities and fluidities of the suspended particles, respectively; s is the volume concentration of the suspended particles; N and D are the turbulent viscosity and diffusion coefficients, respectively; q is the diffusion flow of suspended matter, $|\mathbf{q}| = q$; g is the gravitational acceleration; v is the lateral velocity of the suspension liquid; **a** is the relative velocity of the suspended particles, $|\mathbf{a}| = a$; and p is the pressure. Angle brackets denote an averaging sign, and a prime denotes a pulsation sign.

First we will show that variation should be expected in the equation D = N under certain conditions. We use, for the behavior of particles at low concentration, equations which do not affect the velocity field u(x, t) of the "pure" liquid

$$\frac{d}{dt}s = \mathbf{a}\cdot\nabla s, \qquad \operatorname{div}\mathbf{u} = 0, \qquad \frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{1}{\rho_0}\nabla p. \tag{1}$$

Here terms which express the effect of the molecular structure of the liquid are omitted. With the Reynolds method of averaging we represent the functions as sums of average values and pulsations. We multiply the first equation of (1) by the velocity pulsation and the last equation of (1) by the concentration pulsation. By averaging these equations and adding them together we obtain

$$\left(\frac{\partial}{\partial t} + \langle\mathbf{u}\rangle\cdot\nabla\right)\mathbf{q} = -\Pi\cdot\nabla\langle s\rangle - \mathbf{q}\cdot\nabla\langle\mathbf{u}\rangle + \langle\mathbf{u}'\mathbf{a}\cdot\nabla s'\rangle - \frac{1}{\rho_0}\langle s'\nabla p'\rangle - \operatorname{div}\langle\mathbf{u}'\mathbf{u}'s'\rangle, \tag{2}$$

where $q = \langle u's'\rangle$ and $\Pi = \langle u'u'\rangle$. This equation, which represents the balance of the quantity **q** and which determines this value, is similar to the well-known one ([2], p. 296) for the balance of the quantity Π. From Eq. (2) it follows that the expression **q** which does not depend on **a**, can only occur when the term in Eq. (2) proportional to **a** is negligibly small. This can happen if the ratio modulus of the above mentioned term to the first term in the right-hand side of Eq. (2) is much less than unity, and this, in turn, can occur at:

$$\left|\frac{\mathbf{a}\cdot\nabla\mathbf{q}}{\Pi\cdot\nabla\langle s\rangle}\right| \ll 1.$$

For steady lateral diffusion, in which

$$q = a_y\langle s\rangle, \tag{3}$$

the above inequality assumes the required form

$$\frac{a_y^2}{\Pi_{yy}} \ll 1. \tag{4}$$

During violation of Eq. (4), for example owing to rather high values of a, the equation q = q(a) must hold. Since $q \neq -D\cdot\nabla\langle s\rangle$, D = D(a) must hold and consequently $D \neq N$ since $N \neq N(a)$. Thus, the last inequality shows that during violation of Eq. (4) $D \neq N$. However, it is clear that this inequality is not a sufficient condition for validity of the equation D = N (the validity of the latter is confirmed by experiment [1]).

In [1] it was proposed to use an expression linear in a

$$D = N + \frac{a}{g}\Pi. \tag{5}$$

We will determine the applicability of this expression. For this purpose we will consider the well-known equation [3] for steady lateral balance of lateral turbulence intensity in a uniform two-dimensional suspension flow. As before, we neglect the effect of molecular structure:

$$\frac{1}{2}\frac{d}{dy}\langle\rho v'^2 v\rangle + (\langle\rho\rangle\langle v'^2\rangle + \langle\rho' v'\rangle\langle v\rangle + \langle\rho' v'^2\rangle)\frac{d}{dy}v = (\rho_* - \rho_0)qg_y - \left\langle v'\frac{\partial p'}{\partial y}\right\rangle - \rho_0\rho_*\left\langle v'\operatorname{div}\frac{s(1-s)}{\rho}\mathbf{a}a_y\right\rangle. \tag{6}$$

In [1,3] it was shown on the assumption that some of the terms are small that Eq. (6) leads to the expression D(a), but the conditions smallness have not been formulated quantitatively. We will formulate such conditions. From Eq. (6) it follows that the expression q, which is linear in a, will occur if the terms which are quadratic in a are neglected. We take account of the well-known result [4]:

$$\langle v\rangle = -\frac{\rho_* - \rho_0}{\rho_0}a_y\langle s\rangle,$$

with $\langle s\rangle \ll 1$, when a = const, we assume that $\langle s\rangle \ll 1$ in Eq. (6), we denote $\langle v'^2\rangle$ by Π_{yy}, and we write the following conditions for smallness in the quadratic terms:

$$\left|\frac{\langle\rho' v'\rangle\langle v\rangle}{\rho_0\Pi_{yy}}\right| = \left|\frac{\rho_0\langle v\rangle^2}{\rho_0\Pi_{yy}}\right| = \left|\langle s\rangle\frac{\rho_0 - \rho_*}{\rho_0}\right|^2\frac{a_y^2}{\Pi_{yy}} \ll 1, \tag{7}$$

$$\frac{\rho_* a_y^2 dq/dy}{\rho_0\Pi_{yy}d\langle v\rangle/dy} = \left|\frac{\rho_*}{\rho_* - \rho_0}\right|\frac{a_y^2}{\Pi_{yy}} \ll 1. \tag{8}$$

Condition (7) corresponds to smallness of the second term in parentheses on the left-hand side of Eq. (6). Condition 8 corresponds to smallness of part of the last term in Eq. (6). Equation (3) is taken into account in both conditions.

Inequalities (7) and (8) make it possible to determine when the linear expression q(a) should not be used. Since q and D are proportional, these same inequalities show when Eq. (5) should not be used. However, these inequalities are not sufficient conditions for the validity of Eq. (5) (the validity of this equation has been confirmed by experiment [1]).

Thus, it was possible above to obtain conditions (4), (7), and (8), which indicate the region in which it is not possible to use the corresponding expressions for the diffusion coefficient. It is important to note that in the case of diffusion of air bubbles in a liquid, with $\rho_* \ll \rho_0$, experiments completed [1] confirm Eq. (5), and conditions (7) and (8) are fulfilled. At the same time, for other particles, with $\rho_* \approx \rho_0$, condition (8) will not be fulfilled so that here it will not be possible to use Eq. (5). Condition (8) is also extremely rigid for solid particles in a gas, when $\rho_* \gg \rho_0$. In this case conditions (4) and (8) practically coincide so that if it is not possible to use the expression D = N it is also not possible to use expression (5). In practice condition (7) presents an important limitation on the applicability of Eq. (5) for the case of rather high concentrations of suspended particles.

We note that inequalities are obtained above, which limit the applicability of Eq. (5) by conditions imposed on quantity a. There also exist a number of limiting conditions, which depend on the degree of nonstationarity and nonuniformity of the process. To determine them we return to Eq. (2), in which for simplicity we omit the term proportional to a, assuming that it is small. We express the two last terms of this equation with a form linear in q. We interpret the first term as local dissociation of q in the pressure-velocity field, and the second as displacement of q owing to diffusion. We obtain

$$\frac{d}{dt}\mathbf{q} = -\Pi\cdot\nabla\langle s\rangle - \mathbf{q}\cdot\nabla\langle\mathbf{u}\rangle + \frac{\mathbf{q}}{\tau} + c\,\mathrm{div}\,(N\cdot\nabla\mathbf{q}), \tag{9}$$

where τ is the time scale of turbulence and c is a constant on the order of unity. The hypotheses adopted make this equation similar to the well-known equation [5] for the linear invariant Π.

Hence it follows that the linear diffusion law $q \sim \nabla \langle s\rangle$ will only occur for a ratio modulus, small in comparison with unity, of the first, third, and last terms in Eq. (9) to the second or fourth. From Eq. (9) in this case it follows that $q = -\tau\Pi\cdot\nabla\langle s\rangle$, and $\nu\Pi = N$, since for small particles $D = N$. In practice, slowly changing flows usually occur with lateral diffusion, and for these the above-mentioned three conditions have the following form:

$$\left|\left(\frac{\partial}{\partial t}q + u_x\frac{\partial}{\partial x}q\right)\left(\Pi_{yy}\frac{\partial\langle s\rangle}{\partial y}\right)^{-1}\right| \ll 1, \tag{10}$$

$$\left|\left(q\frac{\partial u_y}{\partial y}\right)(q/\tau)^{-1}\right| = \tau\left|\frac{\partial u_y}{\partial y}\right| \ll 1, \tag{11}$$

$$\left|\left(cN_{yy}\frac{\partial^2}{\partial y^2}q\right)\left(\Pi_{yy}\frac{\partial}{\partial y}\langle s\rangle\right)^{-1}\right| \ll 1. \tag{12}$$

By means of Eq. (3) and the equation $D = N$ the last condition for stationary one-dimensional diffusion reduces to

$$ca_y^2/\Pi_{yy} \ll 1. \tag{13}$$

Thus the equation $D = N$ is valid only for fulfillment of conditions (4) and (10)–(12). It is more difficult to formulate conditions for the validity of Eq. (5) in a similar manner, since this would require information on a number of important features, which at present is not available.

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